The study of randomly excited nonlinear dynamical systems forms the focus of this thesis. We discuss two classes of problems: first, the characterization of nonlinear random response of the system before it comes into existence and, the second, assimilation of measured responses into the mathematical model of the system after the system comes into existence. The first class of problems constitutes forward problems while the latter belongs to the class of inverse problems. An outstanding feature of these problems is that they are almost always not amenable for exact solutions. We tackle in the present study these two classes of problems using Monte Carlo simulation tools in conjunction with Markov process theory, Bayesian model updating strategies, and particle filtering based dynamic state estimation methods.

It is well recognized in literature that any successful application of Monte Carlo simulation methods to practical problems requires the simulation methods to be reinforced with effective means of controlling sampling variance. This can be achieved by incorporating any problem specific qualitative and (or) quantitative information that one might have about system behavior in formulating estimators for response quantities of interest. In the present thesis we outline two such approaches for variance reduction. The first of these approaches employs a substructuring scheme, which partitions the system states into two sets such that the probability distribution of the states in one of the sets conditioned on the other set become amenable for exact analytical solution. In the second approach, results from data based asymptotic extreme value analysis are employed to tackle problems of time variant reliability analysis and updating of this reliability. We exemplify in this thesis the proposed approaches for response estimation and model updating by considering wide ranging problems of interest in structural engineering, namely, nonlinear response and reliability analyses under stationary and (or) nonstationary random excitations, response sensitivity model updating, force identification, residual displacement analysis in instrumented inelastic structures under transient excitations, problems of dynamic state estimation in systems with local nonlinearities, and time variant reliability analysis and reliability model updating. We have organized the thesis into eight chapters and three appendices. A resume of contents of these chapters and appendices follows.

In the first chapter we aim to provide an overview of mathematical tools which form the basis for investigations reported in the thesis. The starting point of the study is taken to be a set of coupled stochastic differential equations, which are obtained after discretizing spatial variables, typically, based on application of finite element methods. Accordingly, we provide a summary of the following topics: (a) Markov vector approach for characterizing time evolution of transition probability density functions, which includes the forward and backward Kolmogorov equations, (b) the equations governing the time evolution of response moments and first passage times, (c) numerical discretization of governing stochastic differential equation using Ito-Taylor's expansion, (d) the partial differential equation governing the time evolution of transition probability density functions conditioned on measurements for the study of existing instrumented structures, (e) the time evolution of response moments conditioned on measurements based on governing equations in (d), and (f) functional recursions for evolution of multi-dimensional posterior probability density function and posterior filtering density function, when the time variable is also discretized. The objective of the description here is to provide an outline of the theoretical formulations within which the problems of response estimation and model updating are formulated in the subsequent chapters of the present thesis. We briefly state the class of problems, which are amenable for exact solutions. We also list in this chapter major text books, research monographs, and review papers relevant to the topics of nonlinear random vibration analysis and dynamic state estimation.

In Chapter 2 we provide a review of literature on solutions of problems of response analysis and model updating in nonlinear dynamical systems. The main focus of the review is on Monte Carlo simulation based methods for tackling these problems. The review accordingly covers numerical methods for approximate solutions of Kolmogorov equations and associated moment equations, variance reduction in simulation based analysis of Markovian systems, dynamic state estimation methods based on Kalman filter and its variants, particle filtering, and variance reduction based on Rao-Blackwellization.

In this review we chiefly cover papers that have contributed to the growth of the methodology. We also cover briefly, the efforts made in applying the ideas to structural engineering problems. Based on this review, we identify the problems of variance reduction using substructuring schemes and data based extreme value analysis and, their incorporation into response estimation and model updating strategies, as problems requiring further research attention. We also identify a range of problems where these tools could be applied.

We consider the development of a sequential Monte Carlo scheme, which incorporates a substructuring strategy, for the analysis of nonlinear dynamical systems under random excitations in Chapter 3. The proposed substructuring ensures that a part of the system states conditioned on the remaining states becomes Gaussian distributed and is amenable for an exact analytical solution. The use of Monte Carlo simulations is subsequently limited for the analysis of the remaining system states. This clearly results in reduction in sampling variance since a part of the problem is tackled analytically in an exact manner. The successful performance of the proposed approach is illustrated by considering response analysis of a single degree of freedom nonlinear oscillator under random excitations. Arguments based on variance decomposition result and Rao-Blackwell theorems are presented to demonstrate that the proposed variance reduction indeed is effective.

In Chapter 4, we modify the sequential Monte Carlo simulation strategy outlined in the preceding chapter to incorporate questions of dynamic state estimation when data on measured responses become available. Here too, the system states are partitioned into two groups such that the states in one group become Gaussian distributed when conditioned on the states in the other group. The conditioned Gaussian states are subsequently analyzed exactly using the Kalman filter and, this is interfaced with the analysis of the remaining states using sequential importance sampling based filtering strategy. The development of this combined Kalman and sequential importance sampling filtering method constitutes one of the novel elements of this study. The proposed strategy is validated by considering the problem of dynamic state estimation in linear single and multi-degree of freedom systems for which exact analytical solutions exist.

In Chapter 5, we consider the application of the tools developed in Chapter 4 for a class of wide ranging problems in nonlinear random vibrations of existing systems. The nonlinear systems considered include single and multi-degree of freedom systems, systems with memoryless and hereditary nonlinearities, and stationary and nonstationary random excitations. The specific applications considered include nonlinear dynamic state estimation in systems with local nonlinearities, estimation of residual displacement in instrumented inelastic dynamical system under transient random excitations, response sensitivity model updating, and identification of transient seismic base motions based on measured responses in inelastic systems. Comparisons of solutions from the proposed substructuring scheme with corresponding results from direct application of particle filtering are made and a satisfactory mutual agreement is demonstrated.

We consider next questions on time variant reliability analysis and corresponding model updating in Chapters 6 and 7, respectively. The research effort in these studies is focused on exploring the application of data based asymptotic extreme value analysis for problems on hand. Accordingly, we investigate reliability of nonlinear vibrating systems under stochastic excitations in Chapter 6 using a two-stage Monte Carlo simulation strategy. For systems with white noise excitation, the governing equations of motion are interpreted as a set of Ito stochastic differential equations. It is assumed that the probability distribution of the maximum over a specified time duration in the steady state response belongs to the basin of attraction of one of the classical asymptotic extreme value distributions. The first stage of the solution strategy consists of selection of the form of the extreme value distribution based on hypothesis testing, and, the next stage involves the estimation of parameters of the relevant extreme value distribution. Both these stages are implemented using data from limited Monte Carlo simulations of the system response. The proposed procedure is illustrated with examples of linear/nonlinear systems with single/multiple degrees of freedom driven by random excitations. The predictions from the proposed method are compared with the results from large scale Monte Carlo simulations, and also with the classical analytical results, when available, from the theory of out-crossing statistics. Applications of the proposed method for vibration data obtained from laboratory conditions are also discussed.

In Chapter 7 we consider the problem of time variant reliability analysis of existing structures subjected to stationary random dynamic excitations. Here we assume that samples of dynamic response of the structure, under the action of external excitations, have been measured at a set of sparse points on the structure. The utilization of these measurements in updating reliability models, postulated prior to making any measurements, is considered. This is achieved by using dynamic state estimation methods which combine results from Markov process theory and Bayes' theorem. The uncertainties present in measurements as well as in the postulated model for the structural behaviour are accounted for. The samples of external excitations are taken to emanate from known stochastic models and allowance is made for ability (or lack of it) to measure the applied excitations. The future reliability of the structure is modeled using expected structural response conditioned on all the measurements made. This expected response is shown to have a time varying mean and a random component that can be treated as being weakly stationary. For linear systems, an approximate analytical solution for the problem of reliability model updating is obtained by combining theories of discrete Kalman filter and level crossing statistics. For the case of nonlinear systems, the problem is tackled by combining particle filtering strategies with data based extreme value analysis. The possibility of using conditional simulation strategies, when applied external actions are measured, is also considered. The proposed procedures are exemplified by considering the reliability analysis of a few low dimensional dynamical systems based on synthetically generated measurement data. The performance of the procedures developed is also assessed based on limited amount of pertinent Monte Carlo simulations.

A summary of the contributions made and a few suggestions for future work are presented in Chapter 8.

The thesis also contains three appendices. Appendix A provides details of the order 1.5 strong Taylor scheme that is extensively employed at several places in the thesis. The formulary pertaining to the bootstrap and sequential importance sampling particle filters is provided in Appendix B. Some of the results on characterizing conditional probability density functions that have been used in the development of the combined Kalman and sequential importance sampling filter in Chapter 4 are elaborated in Appendix C.